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A NOTE ON WIND GENERATOR INTERACTION

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CONTENTS

1. WAKE MODEL

If we neglect the near field behind a wind generator, where periodic and deterministic swirling vortices make a special contribution, it might be possible to treat the resulting wake behind the generator as a turbulent wake or negative jet. The areal spread of momentum deficit of a feature like this is such that linear dimension (radius r) is proportional to downwind distance, x. A balance of momentum gives (see Fig. 1)

$$\pi r_o^2 v_o + \pi(r^2 - r_o^2)u = \pi r^2 v \ , \qquad (1)$$

where v_o is the velocity just behind the rotor, u is the ambient wind velocity, and v is the velocity in the wake at distance x from the generator. Solving for v, and further assuming a linear

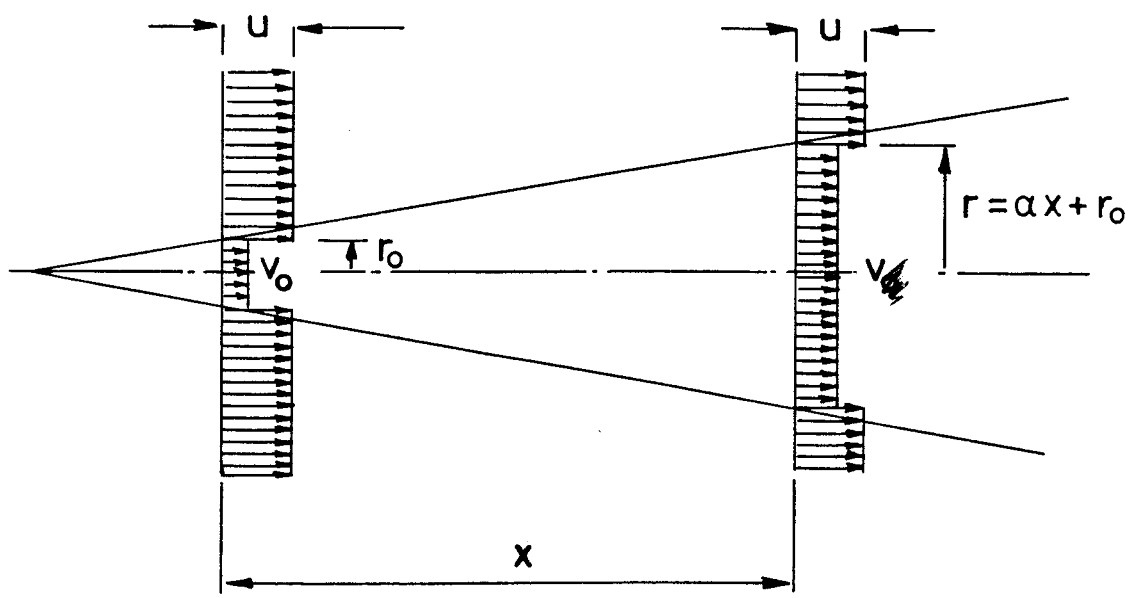

Fig. 1. Schematic wake model with definition of symbols used in the text.

wake ($r \propto x$), and that the velocity just behind the rotor is $\frac{1}{3}u$ in accordance with classical theory, gives

$$v = u\{1 - \frac{2}{3}(\frac{r_o}{r_o+\alpha x})^2\} . \qquad (2)$$

For usual wakes the entrainment constant α is approximately 0.1. Using this value, and relevant values of x and r_o, Eq. (2) is plotted on Fig. 2, where it might be compared to actual measurements by Vermeulen (1979). The agreemnt is seen to be excellent in regard to the observed centerline retardation.

Another source that the simple model can be checked against is the early results from the Nibe-wake project (Høistrup, 1983). Table I compares model prediction and experimental results. Again the agreement is seen to be very satisfactory.

Table I. Wind measurements (approximately 1 hr averages) in the center of the wake of Nibe-A (Høistrup 1983), which has a rotor radius of r = 20 m. The wind speed 100 m upwind was reported to u = 8.10 m/s. The numbers given under the heading "Model" is results from Eq. (2) with α =0.1. Using the measurements to calibrate α produces α = 0.070. However, this refinement is probably not justified as the assumption of $\frac{1}{3}u$ just behind the rotor is fairly uncertain and in reality is dependent on the loading of the turbine.

Downwind distance [m]	Wind velocity in the wake Observed [m/s]	Model [m/s]
40	3.95	4.35
100	5.03	5.70

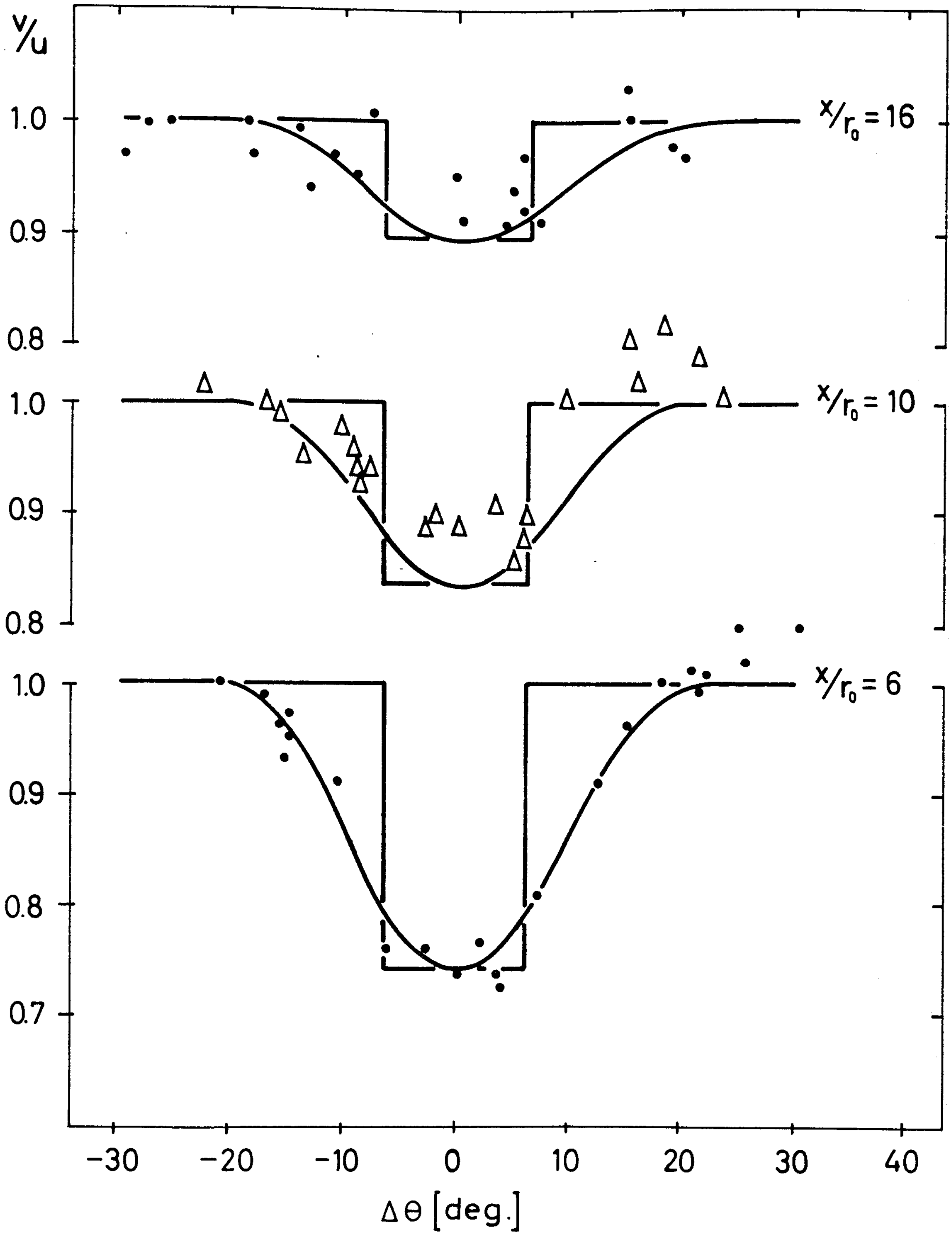

Fig. 2. Measured wake profiles (256 sec average values) after Vermeulen et al. (1979), along with predictions according to Eq. (2) with and without a cosine-bell (Eq. (3)) employed.

Regarding the cross-wind variation, the present simple model prescribes a "top-hat" distribution while the real distribution appears to be more Gaussian or bell-shaped. The difference is shown in Fig. 2, where the width of the bell-shape is taken from Vermeulen (1979), and the width of the top-hat corresponds to $\alpha = 0.1$ as used above. In a climatological appraisal of the wake effect on an other object (e.g. another wind generator) this difference is probably not too important as the regions of over- and under-estimation may cancel, but of course it is possible to apply a modulation such as

$$f(\theta) = \frac{1+\cos(9\theta)}{2} \quad ; \qquad \theta \leq 20^{\circ} \ , \tag{3}$$

to be applied onto the x and r_o dependent part of Eq. (2) to comprise a deficit we shall call the wake function. Here θ is the off-axis angle in degrees. For $\theta \geq 20^{\circ}$ of course $f(\theta) = 0$. In the example to follow in the next section we shall apply this. To show its degree of fit to reality it is plotted on top of Vermeulen's results in Fig. 2.

Further, in the above simple model it is assumed that the wake expands freely (the presence of the solid earth surface is neglected). This causes an overestimated rate of recovery of velocity in the wake. If a more realistic, and pessimistic, estimation of the development of the wake is wanted, it is possible to incorporate it by using an imaginary wind generator at the negative hub height of the actual generator. This technique is well-known in calculations of stack plume development. However, in the present analytical treatment this slight complication is avoided.

2. EXAMPLE WITH 10 GENERATORS IN A CIRCLE

The immediate reaction to a proposition for a number of generators to be put in a circular array is that it might not be beneficial. Of course, it all depends on the size of the generators (wing length or rotor radius) and the diameter of the array. Here we shall examine the output of 10 generators with $r = 10$ m, placed on circle of diameter $D = 200$ m as a function of wind direction. The problem with a circular array is that some generator will always be in the lee in contrast to a simple line of generators which will only interact when the wind is in a certain sector (to be arranged as the less frequent).

Fig. 3 shows a schematic outline of the array in question. Wakes corresponding to Eq. (2) are shown as cross hatched areas. If the wind direction corresponds to the direction of the A-B sector, which has a length of ~ 60 m, the B and F generators will have a much reduced output, and in addition the C, D and E generators will have a slightly reduced output too. According to Eq. (2) the wind velocity at B and F will be approximately 0.74u while at C, D and E it will be about 0.93u. Thus the total output E relative to the output of 10 non-interacting generators E_o may be estimated as

$$\frac{E}{E_o} = \frac{5u^3 + 2(0.74u)^3 + 3(0.93u)^3}{10u^3} \approx 0.82 \; . \qquad (4)$$

This reduction will prevail for a range of wind directions corresponding to the spreading angle of the wake. At a wind direction $\theta = 18^o$ (see Fig. 3) C and F will be in the wake of A and H, and the wind velocity at C and F will be approximately 0.86u. In this situation we have

$$\frac{E}{E_o} = \frac{6u^3 + 2(0.86u)^3 + 2(0.93u)^3}{10u^3} \approx 0.89 \; . \qquad (5)$$

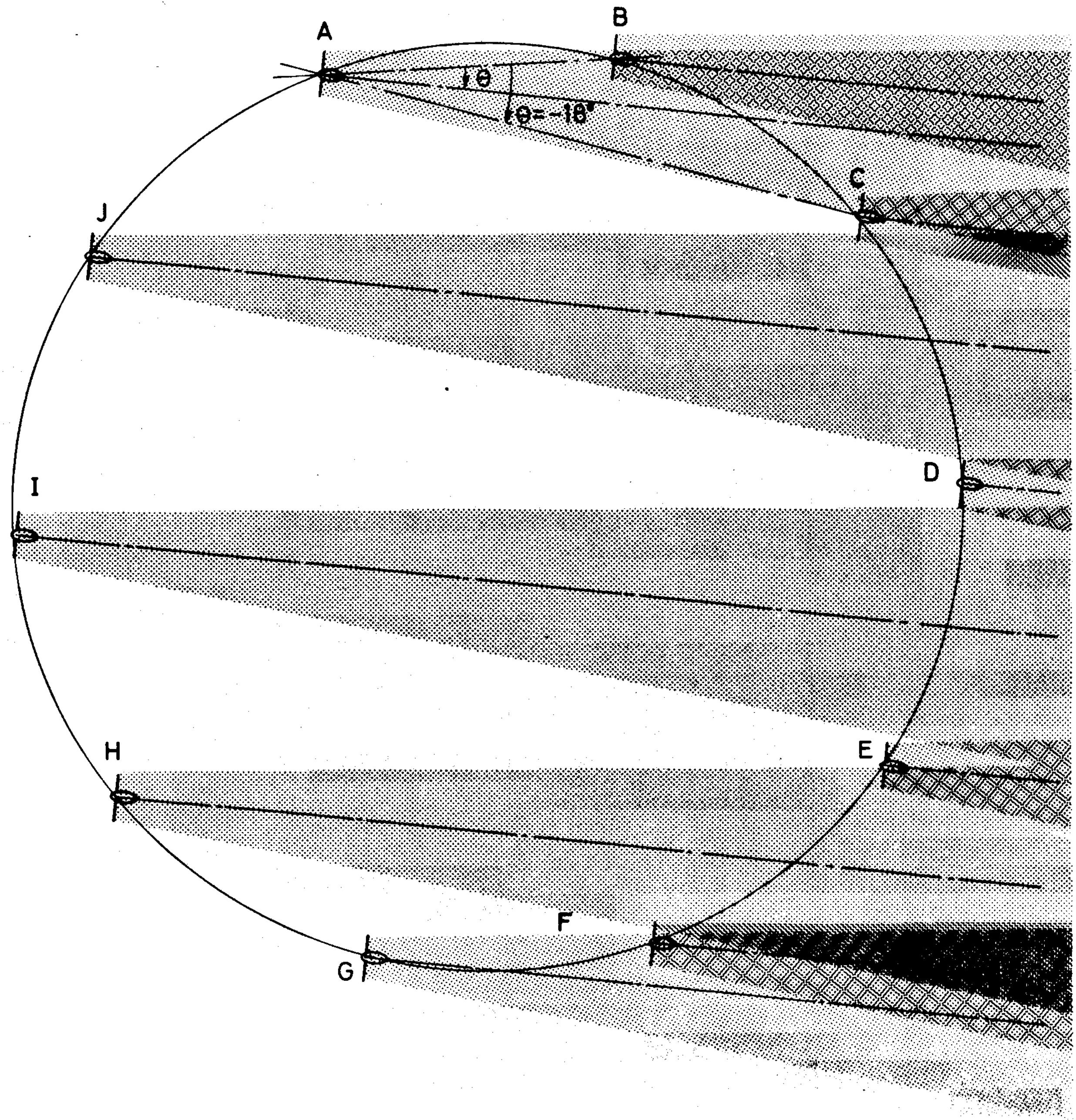

Fig. 3. Schematic outline of a circular array of 10 turbines, with definition of wake direction relative to array orientation.

For a further change in direction, C will gradually become directly in the wake of B, and the situation will then become equivalent to the situation first described. It is easily seen that the problem is symmetric with a period of 36°.

Although the above simplified calculations give a good idea of the order of magnitude of the reduction of the production to be expected from a circular array, a more precise calculation is warranted. Such a calculation is performed here in which the bell shape given by Eq. (3) is used and the calculation is done for steps of 1^{o} in the wind direction. For certain wind directions it happens that the fringes of different wakes overlap at generator sites downstream. In such cases the wake functions are added (in analogy with two plumes of passive tracer where the overlapping region is the sum of the concentrations of the two single plumes, calculated as if the other was not present). The relative energy production thus calculated is shown in Fig. 4 as a function of wind direction. Results are only shown for a range of 36^{o}; beyond this range the curve repeats itself. The level of E/E_o is seen to agree well with the above estimates. The average reduction can be computed to be 0.83.

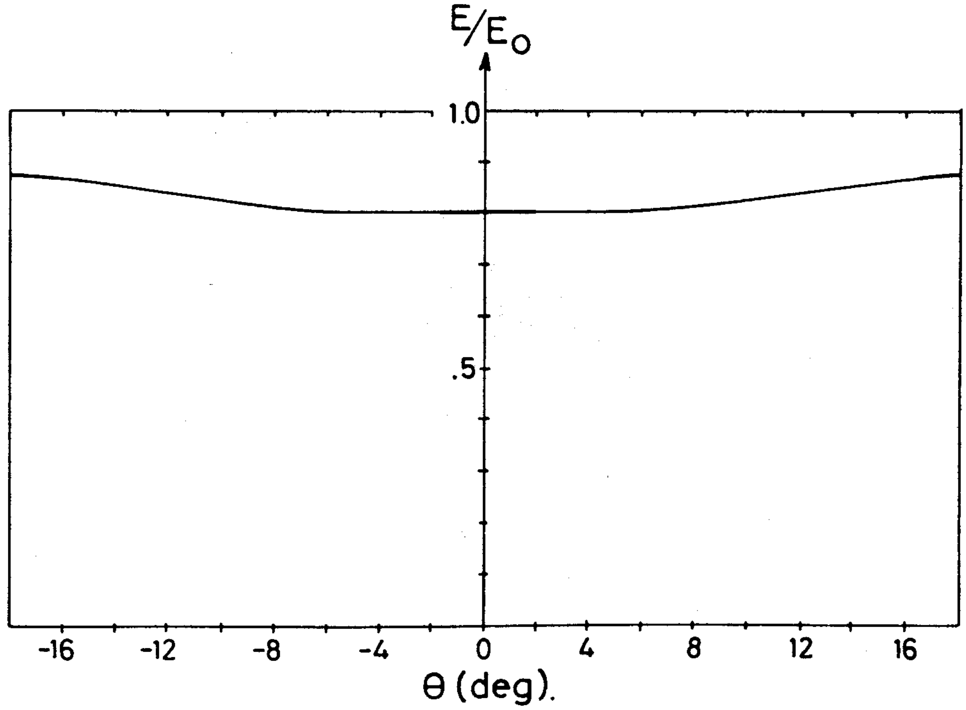

Fig. 4. Calculated output of the circular array mentioned in the text, relative to the output of an equal number of non-interacting generators, given as a function of the wake direction defined in Fig. 3.

3. MULTIPLE WAKES

The above wake model can be applied to the case in which multiple wake interaction occurs, i.e. the case in which a wind generator is in the wake of a generator, which in turn is in the wake of another and so forth. Such an arrangement is pictured in Fig. 5 along with a definition of essential variables. The velocity v_1 in front of generator 2 is given directly by Eq. (2). To find the velocity v_2 in front of generator 3 we apply a balance similar to Eq. (1) immediately using that the velocity directly behind generator 2 is $\frac{1}{3}v_1$:

$$r_0^2(\tfrac{1}{3}v_1) + (r^2-r_0^2)\bar{v}_1 = r^2v_2 \,, \tag{6}$$

where $\bar{v}_1$ is the weighted velocity of air entrained from wake 1 into wake 2 en route from generator 2 to generator 3, and r is the radius of the wake from generator 2 at genrator 3.

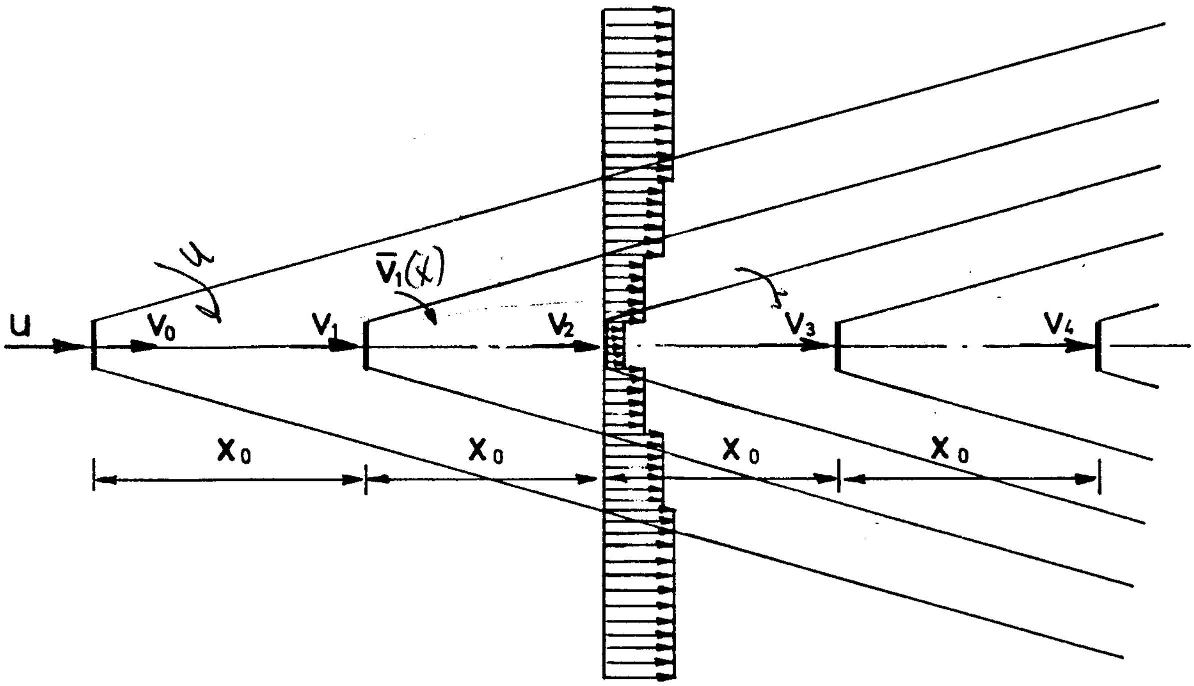

Fig. 5. Schematic multiple wake model with definition of symbols used in the text.

In the spirit of the present model it is possible to compute $\bar{v}_1$ exactly, but the expression is rather complicated and does not lend itself to the further calculations we intend to do. The results of such calculations are that $\bar{v}_1$ is equal to u less a small amount on the order of $0.3(r_o/\alpha x_o)^2 u$, which shows that as distance becomes large compared to rotor dimension, the small fraction might be neglected. In any case, assuming $\bar{v}_1 = u$ is conservative in the sense that it will not underestimate the output of the linear array (the entrainment of momentum is assumed slightly larger than in reality). Thus solving Eq. (6) for v_2 gives approximatley

$$\frac{v_2}{u} = 1 - (1 - \frac{1}{3}\frac{v_1}{u})(\frac{r_o}{r_o+\alpha x_o})^2 \quad , \tag{7}$$

which is seen to be analogous to Eq. (2): the factor 2/3 = $(1 - \frac{1}{3})$ has been substituted by $1 - \frac{1}{3}(v_1/u)$. It is easy to show that in general, to the same approximation,

$$\frac{v_N}{u} = 1 - (1 - \frac{1}{3}\frac{v_{N-1}}{u})(\frac{r_o}{r_o+\alpha x_o})^2 \quad . \tag{8}$$

Introducing the following simplification in notation:

$$\left.\begin{aligned} y_N &= v_N/u \\ k &= (r_o/(r_o+\alpha x_o))^2 \end{aligned}\right\} \quad , \tag{9}$$

we get

$$y_N = 1 - k(1 - \tfrac{1}{3}y_{N-1}) \quad , \tag{10}$$

which means that

$$y_1 = 1 - k\tfrac{2}{3} \ ;$$
$$y_2 = 1 - k\tfrac{2}{3} - k^2\tfrac{2}{9} \ ;$$
$$y_3 = 1 - k\tfrac{2}{3} - k^2\tfrac{2}{9} - k^3\tfrac{2}{27} \ ;$$

and in general

$$y_N = y_{n-1} - 2(\tfrac{k}{3})^N \ . \qquad (11)$$

Combination of Eqs. (10) and (11) gives the explicit expression

$$y_N = 1 - 2\frac{k}{3}\,\frac{1-(k/3)^N}{1-k/3} \ .$$

As k is of order unity or less, the term $(k/3)^N$ vanishes very quickly with N increasing, and we get the asymptotic expression

$$\frac{v_\infty}{u} = 1 - \frac{2x}{1-x} \quad ; \quad x \equiv \frac{1}{3}\left(\frac{r_o}{r_o+\alpha x_o}\right)^2 \ . \qquad (12)$$

4. EXAMPLE WITH 10 GENERATORS IN A ROW

Let us again choose a rotor radius of r_o = 10m. The asymptotic value of velocity in front of the downwind generators of course depends on the distance x_o between generators in the array: For x_o = 50m, $v_\infty \cong 0.65u$ and for x_o = 100m, $v_\infty \cong 0.82u$. In fact already in front of the third generator the wind is down to these values, while in front of the second the speed is 0.70u and 0.83u for x_o = 50 and 100 m respectively. Thus the energy estimates for these two arrays relative to undisturbed generators are

$$\frac{E}{E_o} = \begin{cases} \dfrac{u^3 + (0.70u)^3 + 8(0.65u)^3}{10u^3} = 0.35 \quad ; \quad x_o/r_o = 5 \\[2ex] \dfrac{u^3 + (0.83u)^3 + 8(0.82u)^3}{10u^3} = 0.60 \quad ; \quad x_o/r_o = 10 \end{cases} .$$

Hence it is realized that the dependence on x_o is quite marked. However, this is only of importance in a quite narrow (α=0.1 corresponds to 12°) wind direction sector; for other wind directions the generators are not interacting.

To be sure of not favouring the linear array above the circular, let us say that the whole of one standard wind sector of 30° is affected. If the wind rose is assumed to be circular, the average output becomes

$$\left\langle\frac{E}{E_o}\right\rangle = \begin{cases} \dfrac{10 + 2\cdot 0.35}{12} \sim 0.89 \\[2ex] \dfrac{10 + 2\cdot 0.60}{12} \sim 0.93 \end{cases}$$

for the two sets of distances. The thing t note here is that x_o is not a very important parameter when it comes to the average output, because the generators are not interacting for most of the time.

In the case where the generators are only 50 m apart, the power production is seen to be only ~ 10% down, even with the angle of the affected sector overestimated a factor of order two (30° instead of 12°). But, the point I finally want to make is that 10% is an overestimate also for another reason: prevailing wind direction. Thus in Denmark norhterly and southerly winds are only half as frequent as westerly winds, which put another factor of 2 decrease on the 10% figure.

5. REFERENCES

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